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Full paper

On the first principle theory of nanogenerators from Maxwell's equations

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ABSTRACT

Nanogenerators (NGs) are a field that uses Maxwell's displacement current as the driving force for effectively converting mechanical energy into electric power/signal, which have broad applications in energy science, environmental protection, wearable electronics, self-powered sensors, medical science, robotics and artificial intelligence. NGs are usually based on three effects: piezoelectricity, triboelectricity (contact electrification), and pyroelectricity. In this paper, a formal theory for NGs is presented starting from Maxwell's equations. Besides the general expression for displacement vector $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ used for deriving classical electromagnetic dynamics, we added an additional term $\boldsymbol{P_s}$ in $\boldsymbol{D}$, which represents the polarization created by the electrostatic surface charges owing to piezoelectricity and/or triboelectricity as a result of mechanical triggering in NG. In contrast to $\boldsymbol{P}$ that is resulted from the electric field induced medium polarization and vanishes if $\boldsymbol{E} = 0$, $\boldsymbol{P_s}$ remains even when there is no external electric field. We reformulated the Maxwell equations that include both the medium polarizations due to electric field ($\boldsymbol{P}$) and non-electric field (such as strain) ($\boldsymbol{P_s}$) induced polarization terms, from which, the output power, electromagnetic behavior and current transport equation for a NG are systematically derived. A general solution is presented for the modified Maxwell equations, and analytical solutions about the output potential are provided for a few cases. The displacement current arising from $\varepsilon \partial \mathbf{E}/\partial t$ is responsible for the electromagnetic waves, while the newly added term $\partial \boldsymbol{P_s}/\partial t$ is the application of Maxwell's equations in energy and sensors. This work sets the first principle theory for quantifying the performance and electromagnetic behavior of a nanogenerator in general.

1. Introduction

Maxwell's equations are among the top 10 most important equations for physics. Ever since their first introduction in 1861 and theoretical prediction about the existence of electromagnetic (EM) wave, especially after the first experimental observation of EM wave in 1886 by Hertz, the Maxwell's equations are the foundation of modern wireless communication, photonics, light communication and many more. Their vast applications cover almost every corner of our life. The theory of EM waves is a direct result of Maxwell's equations, which is our general understanding about their practical implications. One of the most greatest creative ideas by Maxwell in 1861 was the introduction of displacement current, $\partial \mathbf{D}/\partial t$, in the Ampere's law, in order to satisfy the conservation law of charges, which resulted in the unification of electricity and magnetism, where $\boldsymbol{D}$ is called the electric displacement vector, based on which Maxwell proves the equivalence of electricity and magnetism. Our general understanding is that the Maxwell equations are the theory for EM wave and light, so that it is most well known in communication and optics sciences. Recently, it has been expanded to calculate the power output of nanogenerators.

Nanogenerators (NGs) are mainly based on three effects: piezoelectricity, triboelectricity, and pyroelectricity. It is called nano-generator because it was first introduced when using a single ZnO nanowire as triggered by the tip of an atomic force microscope for converting tiny mechanical energy into electric power. But with the further physics understanding and development of the field, *NGs are now referred as a field that uses displacement current as the driving force for effectively converting mechanical energy into electric power/signal, disregard if nanomaterials are used or not*. The first piezoelectric NG (PENG) was invented in 2006 [1], and the first triboelectric NG (TENG) was invented in 2012 [2]. As of now, research in NGs has attracted worldwide interest owing to their applications as micro/nano-power sources, self-powered sensors, harvesting blue energy and high-voltage sources [3]. NGs are referred as the energy technology for the new era –the era of internet of things [4].

In 2017, Wang first expanded the expression of displacement current

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and first introduced a term $\boldsymbol{P}_s$ in $\boldsymbol{D}$ for deriving the output power of NGs [5,6], where $\boldsymbol{P}_s$ is *the polarization created by the electrostatic surface charges owing to mechanical triggering, which is different from that the of electric field induced medium polarization* $\boldsymbol{P}$. Such charges are from piezoelectric polarization and triboelectrification regardless if there is externally applied electric field or not. As a result, the theory for NGs has been set from the first principle point of view. Recently, using the theory of displacement current, theoretical calculations for different modes of TENGs have well developed for explaining the observed power output and the observation of wireless power transmission using displacement current.

In this paper, starting from the Maxwell equations, we formally introduce the $\boldsymbol{P}_s$ term in both the Ampere's law and the Gauss's theorem, and derive a new set of Maxwell equations that serves as the first principle theory for quantifying the output and its electromagnetic behavior of NGs. A general solution will be presented in the integral form that is expected to be applicable for any NG cases as long as the distribution function of the electrostatic charges on the surfaces is provided. The theory not only can derive the output power of the NGs, but also can predict the electromagnetic radiation from NGs. We will discuss the relationships between the displacement current and the experimentally observed capacitive conduction current across a load. The objective of this paper is to set the fundamental theoretical frame for NGs for a general case.

2. Physics picture and theoretical frame

2.1. General equations

We first start from the Maxwell's equations:

$$\nabla\cdot\boldsymbol{D} = \rho \tag{1.1}$$

$$\nabla\cdot\boldsymbol{B} = 0 \tag{1.2}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \tag{1.3}$$

$$\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial\boldsymbol{D}}{\partial t} \tag{1.4}$$

Note that ρ in Eq. (1.1) is the distribution of free charges in space, $\boldsymbol{J}$ in Eq. (1.4) is the density of free conduction current density in space as a result of charge flow, and $\boldsymbol{D}$ is called the electric displacement vector, $\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$. We know that the effect of polarization with the presence of electric field is to produce accumulations of bound charges, $\rho_b = -\nabla\cdot\boldsymbol{P}$ within the volume of the media and $\sigma_b = -\boldsymbol{P}\cdot\mathbf{n}$ on the media surface, where the $\boldsymbol{P}$ is the medium polarization vector and $\mathbf{n}$ is the unit vector of the normal direction of the surface. The field due to polarization of the medium is just the field of the bounded charges.

In general, with the presence of electric field $\boldsymbol{E}$, the dielectric will be polarized and $\boldsymbol{P}$ is expressed as $\boldsymbol{P} = (\varepsilon - \varepsilon_0)\boldsymbol{E}$ for an isotropic dielectric medium, which is a result of electric field induced medium polarization. In general, $\boldsymbol{P}$ vanishes if $\boldsymbol{E} = 0$. Thus, $\boldsymbol{D} = \varepsilon\boldsymbol{E}$, which means that there is no displacement current if there is no electric field ($\boldsymbol{E} = 0$), or there is no polarization if there is no external electric field. This is the general case for EM wave, and all of the theories and applications have been developed for this case [7].

However, in practice, polarization can also be produced by strain field as a result of piezoelectric effect and surface contact-electrification (e.g., triboelectric effect), which is independent of the presence of electric field. In the piezoelectric case, surface polarization charges are created due to the strain induced ions on crystal surfaces. In the case of TENGs, triboelectric charges are produced on surfaces simply due to a physical contact between two different materials. To account for the contribution made by the contact electrification induced electrostatic charges in the Maxwell's equations, an additional term $\boldsymbol{P}_s$ is added in $\boldsymbol{D}$ by Wang in 2017 [5,6], that is

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} + \boldsymbol{P}_s \tag{2.1}$$

Here, the first term polarization vector $\boldsymbol{P}$ is due to the existence of an external electric field, and the added term $\boldsymbol{P}_s$ *is mainly due to the existence of the surface charges that are independent of the presence of electric field.* Substituting Eq. (2.1) into Maxwell's equations, and define

$$\boldsymbol{D}' = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} \tag{2.2}$$

then, we can write the Maxwell's equations as follows

$$\nabla\cdot\boldsymbol{D}' = \rho - \nabla\cdot\boldsymbol{P}_s \tag{3.1}$$

$$\nabla\cdot\boldsymbol{B} = 0 \tag{3.2}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \tag{3.3}$$

$$\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial\boldsymbol{D}'}{\partial t} + \frac{\partial\boldsymbol{P}_s}{\partial t} \tag{3.4}$$

Therefore, the volume charge density and the density of current density can be redefined as

$$\rho' = \rho - \nabla\cdot\boldsymbol{P}_s \tag{4.1}$$

$$\boldsymbol{J}' = \boldsymbol{J} + \frac{\partial\boldsymbol{P}_s}{\partial t} \tag{4.2}$$

which satisfies the charge conversation and continuation equation:

$$\nabla\cdot\boldsymbol{J}' + \frac{\partial\rho'}{\partial t} = 0 \tag{4.3}$$

From the above, the Maxwell's equations Eqs. (1.1)–(1.4) can be rewritten as the new set of four self-consistent equations:

$$\nabla\cdot\boldsymbol{D}' = \rho' \tag{5.1}$$

$$\nabla\cdot\boldsymbol{B} = 0 \tag{5.2}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \tag{5.3}$$

$$\nabla\times\boldsymbol{H} = \boldsymbol{J}' + \frac{\partial\boldsymbol{D}'}{\partial t} \tag{5.4}$$

These equations are the corner stones for deriving the output characteristics of NGs.

2.2. Equations for potential functions

Given the distribution of free charges $\rho(\mathbf{r}, t)$ and conduction current $\boldsymbol{J}(\mathbf{r}, t)$, we can calculate the fields $\boldsymbol{E}(\mathbf{r}, t)$ and $\boldsymbol{B}(\mathbf{r}, t)$. The concept of the vector magnetic potential $\boldsymbol{A}$ was introduced because of the solenoidal nature of $\boldsymbol{B}$ ($\nabla\cdot\mathbf{B} = \mathbf{0}$):

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} \tag{6.1}$$

To be consistent with the definition of the scalar electric potential ϕ for electrostatics, we define

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial\boldsymbol{A}}{\partial t} \tag{6.2}$$

Substitute Eqs. (6.1) and (6.2) into Eq. (5.4) and make use of the constitutive relations, we have,

$$\nabla^2\boldsymbol{A} - \mu\varepsilon\frac{\partial^2\boldsymbol{A}}{\partial t^2} = -\mu\boldsymbol{J}' + \nabla\left(\nabla\cdot\boldsymbol{A} + \mu\varepsilon\frac{\partial\phi}{\partial t}\right) \tag{6.3}$$

Using Lorentz gauge,

$$\nabla\cdot\boldsymbol{A} + \mu\varepsilon\frac{\partial\phi}{\partial t} = 0 \tag{6.4}$$

which makes the second term on the right-hand side of Eq. (6.3) vanish, so we obtain

$$\nabla^2\boldsymbol{A} - \mu\varepsilon\frac{\partial^2\boldsymbol{A}}{\partial t^2} = -\mu\boldsymbol{J}' \tag{7.1}$$

This is a nonhomogeneous wave equation for vector potential $\boldsymbol{A}$. It is called a wave equation because its solutions represent waves traveling with a velocity equal to $c = 1/\sqrt{\mu\varepsilon}$.

A corresponding wave equation for the scalar potential ϕ can be obtained by substituting Eq. (6.2) in Eq. (5.1), we have

$$\nabla^2\phi - \mu\varepsilon\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho'}{\varepsilon} \tag{7.2}$$

which is a nonhomogeneous wave equation for scalar potential ϕ. Once the solution of $\boldsymbol{A}$ and ϕ can be found, the total electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ can be calculated.

2.3. Wave equations and their integral solutions

The new form of the Maxwell's equations gives a complete description of the relation between electromagnetic fields and charges and current distributions for nanogenerators. In fact, for given free charge and current distributions, ρ and $\boldsymbol{J}$, we first solve the nonhomogeneous wave equations, Eqs. (7.1) and (7.2), for potential $\boldsymbol{A}$ and ϕ. With $\boldsymbol{A}$ and ϕ determined, $\boldsymbol{B}$ and $\boldsymbol{E}$ can be found from Eqs. (6.1) and (6.2), respectively.

Now consider the solutions of Eq. (7.2) for scalar electric potential ϕ first We can do this by first finding the solution for a point charge located at $\boldsymbol{r}$' at time t, which can be done using the definition of Green's function and delta function. Then sum up the contributions made by all of the charges distributed in space. The distance from the source point $\boldsymbol{r}$' to the field point $\boldsymbol{r}$ is defined as $\Delta\boldsymbol{r}$, which causes a time delay,

$$t' = t - |\boldsymbol{r} - \boldsymbol{r}'|/c \tag{8.1}$$

Follow the standard procedures as presented in Jackson's text book on electrodynamics book [see chapter 6 in Ref. [7]], using the definition of Green function and delta function, the integral form of the potential due to a charge distribution over a volume is given by

$$\phi(\boldsymbol{r},\ t) = \frac{1}{4\pi\varepsilon}\int\frac{\rho'\left(\boldsymbol{r}',\ t - \left|\frac{r-r'}{c}\right|\right)}{|\boldsymbol{r} - \boldsymbol{r}'|}d\boldsymbol{r}' \tag{8.2}$$

Here the ρ' ($\boldsymbol{r}$', t) is the charge density that prevailed at point $\boldsymbol{r}$' at the retarded time t' as given in Eq. (8.1). Because the integrals are evaluated at the retarded time, these are called retarded potentials. The solution of Eq. (7.1), for vector magnetic potential $\boldsymbol{A}$, can proceed in exactly the same way as that for ϕ. The retarded vector potential is thus given by

$$\mathbf{A}(\boldsymbol{r},\ t) = \frac{\mu}{4\pi}\int\frac{\boldsymbol{J}'\left(\boldsymbol{r}',t - \left|\frac{r-r'}{c}\right|\right)}{|\boldsymbol{r} - \boldsymbol{r}'|}d\boldsymbol{r}' \tag{8.3}$$

As stated before, when the ϕ and $\boldsymbol{A}$ are determined, we can obtain the $\boldsymbol{B}$ and $\boldsymbol{E}$ from Eqs. (6.1) and (6.2), respectively. Both Eqs. (8.2) and (8.3) satisfy the Lorentz condition Eq. (6.4).

If the NG works in a relatively high frequency range, such as to MHz, the electromagnetic radiation of NG can be calculated using the above equations. Then, Poynting vector is

$$\boldsymbol{S} \cong \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) \tag{9.1}$$

And the radiated power of TENGs that can be detected by a detector is

$$\boldsymbol{P} = \int \boldsymbol{S}\cdot d\boldsymbol{s} \tag{9.2}$$

which means that if one gets the $\boldsymbol{B}$ and $\boldsymbol{E}$ from Eqs. (6.1) and (6.2), he can calculate the total radiated power from NGs based on Eq. (9.2). Such experiments could be possible with using high frequency excitation. But for general TENGs, electromagnetic radiation can be ignored.

3. Theory for nanogenerators

3.1. General theory for potential and fields

A nanogenerator is made of dielectric media that produces the strain induced surface electrostatic charges on surfaces, which is responsible for the $\boldsymbol{P}_s$ term, the electrodes that have the free charge distribution ρ, and interconnect conductive wire across the external load that carries the free-flowing current ($\boldsymbol{J}$). From Eqs. (4.1) and (4.2), the key to the calculation is $\boldsymbol{P}_s$, the calculation of which is illustrated as follows. If the surface charge density function $\sigma_s(\boldsymbol{r},t)$ on the surfaces of the media is defined by a shape function of $f(\boldsymbol{r},t) = 0$, where the time is introduced to represent the instantons shape of the media with considering external triggering (Fig. 1), the equation for defining $\boldsymbol{P}_s$, can be expressed as

$$\nabla\cdot\boldsymbol{P}_s = -\sigma_s(\boldsymbol{r},t)\delta(f(\boldsymbol{r},t)) \tag{10.1}$$

where $\delta(f(\boldsymbol{r},t))$ is a delta function that is introduced to confine the shape of the media $f(\boldsymbol{r},t) = 0$ so that the polarization charges produced by non-electric field are confined on the medial surface, and which is defined as follows:

$$\delta(f(\boldsymbol{r},t))\begin{cases}\infty & if\ f(\boldsymbol{r},t) = 0\\ 0 & otherwise\end{cases} \tag{10.2}$$

$$\int_{-\infty}^{\infty}\delta(f(\boldsymbol{r},t))dn = 1 \tag{10.3}$$

where $\boldsymbol{n}$ is the normal direction of the local surface, and dn is an integral along the surface normal direction of the media. If we define the "potential" induced by $\boldsymbol{P}_s$ by: $\boldsymbol{P}_s$ = - $\nabla\phi_s(\boldsymbol{r},\ t)$, we have

$$\nabla^2\phi_s(\boldsymbol{r},\ t) = \sigma_s(\boldsymbol{r},t)\delta(f(\boldsymbol{r},t)) \tag{10.4}$$

The solution is thus given by

$$\phi_s(\boldsymbol{r},\ t) = \frac{1}{4\pi}\int\frac{\sigma_s(\boldsymbol{r}',\ t')\ \delta(f(\boldsymbol{r},'t'))}{|\boldsymbol{r} - \boldsymbol{r}'|}d\boldsymbol{r}' = \frac{1}{4\pi}\int\frac{\sigma_s(\boldsymbol{r}',\ t')}{|\boldsymbol{r} - \boldsymbol{r}'|}ds' \tag{11.1}$$

where ds' is an integral over the surface $f(\boldsymbol{r},t) = 0$ of the dielectric media (Fig. 1). Therefore, the polarization arising from the surface charge density is

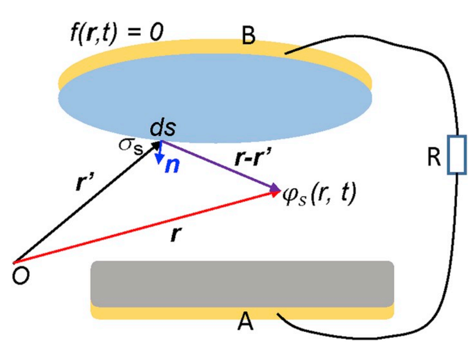

Fig. 1. Schematic representation of a nanogenerator that is hooked up with an external load, and the corresponding coordination system for mathematical description. There four basic components for the NG: two dielectric media of artificial shape, and two electrodes A and B.

$$\boldsymbol{P}_s = -\nabla\phi_s(\boldsymbol{r},\ t) = -\frac{1}{4\pi}\nabla\int\frac{\sigma_s(\boldsymbol{r}^{'},\ t^{'})}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}ds^{'}$$
$$= \frac{1}{4\pi}\int\sigma_s(\boldsymbol{r}^{'},\ t^{'})\frac{\boldsymbol{r}-\boldsymbol{r}^{'}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|^3}ds^{'} + \frac{1}{4\pi c}\int\frac{\partial\sigma_s(\boldsymbol{r}^{''},\ t^{'})}{\partial t^{'}}\frac{\boldsymbol{r}-\boldsymbol{r}^{'}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|^2}ds^{'} \tag{11.2}$$

If we ignore the time delay term for the medial moving speed is rather low compare to the speed of light for NGs, which is the case in practice,

$$\boldsymbol{P}_s \approx \frac{1}{4\pi}\int\sigma_s(\boldsymbol{r}^{'},\ t)\frac{\boldsymbol{r}-\boldsymbol{r}^{'}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|^3}ds^{'} \tag{12.1}$$

$$\frac{\partial\boldsymbol{P}_s}{\partial t} = \frac{1}{4\pi}\int\frac{\partial\sigma_s(\boldsymbol{r}^{'},\ t)}{\partial t}\frac{\boldsymbol{r}-\boldsymbol{r}^{'}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|^3}ds^{'} \tag{12.2}$$

The vector potential due to the surface polarization $\boldsymbol{P}_s$ is

$$\boldsymbol{A}_s(\boldsymbol{r},\ t) \approx \frac{\mu}{4\pi}\int\frac{\frac{\partial\boldsymbol{P}_s(\boldsymbol{r}^{'},\ t)}{\partial t}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}d\boldsymbol{r}^{'}$$
$$= \frac{1}{4\pi}\frac{\mu}{4\pi}\int\int\frac{\partial\sigma_s(\boldsymbol{r}^{''},\ t)}{\partial t}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}\frac{\boldsymbol{r}^{'}-\boldsymbol{r}^{''}}{|\boldsymbol{r}^{'}-\boldsymbol{r}^{''}|^3}ds^{''}d\boldsymbol{r}^{'} \tag{13}$$

For a special case, that ρ_s is a constant on the surface such as the planar geometry for NG in practice, the surface integral is simplified as a solid angle integral, resulting in

$$\boldsymbol{P}_s = \sigma_s(t)\ \mathbf{n} \tag{14.1}$$

$$\frac{\partial\boldsymbol{P}_s}{\partial t} = \frac{\partial\sigma_s(t)}{\partial t}\boldsymbol{n} \tag{14.2}$$

By including the contributions from the free changes and free conduction current, the total electric potential in space is

$$\phi(\boldsymbol{r},\ t) = \frac{1}{4\pi\varepsilon}\int\frac{\rho(\boldsymbol{r}^{'},\ t^{'})}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}d\boldsymbol{r}^{'} + \frac{1}{4\pi\varepsilon}\int\frac{\sigma_s(\boldsymbol{r}^{'},\ t^{'})}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}ds^{'} \tag{15.1}$$

The vector potential is

$$\mathbf{A}(\boldsymbol{r},\ t) = \frac{\mu}{4\pi}\int\frac{\boldsymbol{J}(\boldsymbol{r}^{'},\ t^{'})}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}d\boldsymbol{r}^{'}$$
$$+\frac{1}{4\pi}\frac{\mu}{4\pi}\int\int\frac{\partial\sigma_s(\boldsymbol{r}^{''},\ t^{''})}{\partial t}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}\frac{\boldsymbol{r}^{'}-\boldsymbol{r}^{''}}{|\boldsymbol{r}^{'}-\boldsymbol{r}^{''}|^3}ds^{''}d\boldsymbol{r}^{'} \tag{15.2}$$

where $t'' = t - |\boldsymbol{r}' - \boldsymbol{r}''|/c$. These are the potentials to be used for calculating $\boldsymbol{E}$ and $\boldsymbol{B}$ in space using Eqs. (6.1)–(6.2).

In the case of NG, since the free charges are distributed on a plane, from Eqs. (15.1)–(15.2), we have

$$\phi(\boldsymbol{r},\ t) = \frac{1}{4\pi\varepsilon}\int\frac{\sigma(\boldsymbol{r}^{'},\ t) + \sigma_s(\boldsymbol{r}^{'},\ t^{'})}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}ds^{'} \tag{16.1}$$

$$\mathbf{A}(\boldsymbol{r},\ t) = \frac{\mu}{4\pi}\int\frac{\frac{\partial\sigma(\boldsymbol{r}^{'},\ t)}{\partial t}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}d\boldsymbol{L}^{'}$$
$$+\frac{1}{4\pi}\frac{\mu}{4\pi}\int\int\frac{\partial\sigma_s(\boldsymbol{r}^{''},\ t)}{\partial t}\frac{1}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}\frac{\boldsymbol{r}^{'}-\boldsymbol{r}^{''}}{|\boldsymbol{r}^{'}-\boldsymbol{r}^{''}|^3}ds^{''}d\boldsymbol{r}^{'} \tag{16.2}$$

For the first term in Eq. (16.2), the integral is over the electrodes and conduction line $\boldsymbol{L'}$ connecting the load R (Fig. 1), which is a vector representing the distribution of the electric current in the electrodes and in the interconnecting line.

If the triboelectric charge density reaches a steady state, the second term in (16.2) would drop out, thus

$$\mathbf{A}(\boldsymbol{r},\ t) = \frac{\mu}{4\pi}\int\frac{\frac{\partial\sigma(\boldsymbol{r}^{'},\ t)}{\partial t}}{|\boldsymbol{r}-\boldsymbol{r}^{'}|}d\boldsymbol{L}^{'} \tag{17}$$

3.2. The current transport equation for nanogenerators

From Eqs. (2.1) and (2.2), the total displacement current density is:

$$\boldsymbol{J}_D = \frac{\partial\boldsymbol{D}^{'}}{\partial t} + \frac{\partial\boldsymbol{P}_s}{\partial t} = \varepsilon\frac{\partial\boldsymbol{E}}{\partial t} + \frac{\partial\boldsymbol{P}_s}{\partial t} \tag{18.1}$$

The displacement current is a surface integral of $\boldsymbol{J}_D$:

$$I_D = \int\boldsymbol{J}_D.ds = \int\frac{\partial\boldsymbol{D}}{\partial t}.ds = \frac{\partial}{\partial t}\int\nabla\cdot\boldsymbol{D}\ d\boldsymbol{r} = \frac{\partial}{\partial t}\int\rho\ d\boldsymbol{r} = \frac{\partial Q}{\partial t} \tag{18.2}$$

where Q is the total free charges on the electrode. *This equation means that the internal circuit in NG is dominated by the displacement current (left-hand side in (18.2)), and the observed current in the external circuit is the capacitive conduction current (right-hand side in (18.2)).* Such a description can be illustrated at the right-hand side bottom of Fig. 2. *The internal circuit and external circuit can meet at the two electrodes, forming a complete loop. Therefore, the displacement current is the intrinsic physical core of current generation and it is the internal driving force, and the capacitive conduction current in an external circuit is the external manifestation of displacement current.*

There are two types of currents: conduction current and displacement current (Fig. 2). The condition current is the results of electron flow in conductors. The electromagnetic generator is based on the Lorentz force driven electron flow in conduction wire, which is the major approach for power generation. While for piezoelectric, pyroelectric, triboelectric, electrostatic and electret effects based generators, the current is driven by the displacement current inside the generator. This type of generator is called nanogenerators that physically represents *a field that uses displacement current as the driving force for effectively converting mechanical energy into electric power/signal.* The materials to be used for NGs can contain nanomaterials or not, it does not change the physics meaning of the nanogenerators. The EM generator works effectively at high frequency because of its characteristic of high current but low voltage, while NG works the best at low frequency because of its high output voltage but low current. In low frequency condition, TENG is most effective for converting wasted and low-quality energy in our living environment into electric power. It is due to this character that TENG has key applications for internet of things, sensor networks and many more.

We now derive the current transport in a loop hooking a nanogenerator with an external load R (Fig. 1). In the external circuit of the

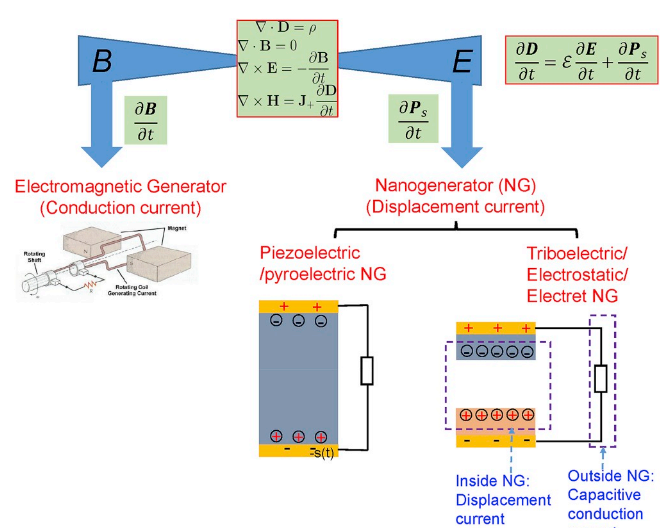

Fig. 2. Schematic showing the conduction current dominated electromagnetic generator and displacement current dominated nanogenerators based on piezoelectric/pyroelectric, triboelectric/electrostatic/electret effects. The difference and relationship between the two are illustrated.

NG from A electrode to B electrode (see Fig. 1), the potential drop around the loop is zero:

$$\phi_{AB} + \phi_{BA} = -\int_A^B \boldsymbol{E}\cdot d\boldsymbol{L} - \oint \frac{\partial \boldsymbol{A}}{\partial t}\cdot d\boldsymbol{L} + I_D R = 0 \quad (19.1)$$

where integral $d\boldsymbol{L}$ is over a path from point A to point B. Since $\oint \frac{\partial \boldsymbol{A}}{\partial t}\cdot d\boldsymbol{L} = \frac{\partial}{\partial t}\int \nabla \times \boldsymbol{A}\cdot d\boldsymbol{s} = \frac{\partial}{\partial t}\int \boldsymbol{B}\cdot d\boldsymbol{s} = 0$, therefore

$$\phi_{AB} = \int_A^B \boldsymbol{E}\cdot d\boldsymbol{L} = \frac{\partial Q}{\partial t}R \quad (19.2)$$

Equation (19.2) is a general equation used to derive the current transport behavior of any configuration of NG. The output power is

$$p = \left(\frac{\partial Q}{\partial t}\right)^2 R \quad (19.3)$$

As for TENG, Eqs. (19.1)-(19.3) have been applied to calculate the power output for contact-separation mode [8,12], lateral-sliding mode [9–11], single-electrode mode [12,13] and free standing mode [14]. A general review on such calculations has been given by Niu et al. [15] Recently, using the distance-dependent model and starting from the displacement current, Shao et al. have calculated the structure figure-of-merits for TENG [16]. Furthermore, numerical calculation has been done for single-electrode mode for 3D configuration [17]. Dharmasena et al. have simulated more complex geometry and results have been compared with experiments [18].

3.3. The output of piezoelectric nanogenerators

As shown in Fig. 3a for a thin film based piezoelectric NG, which is an insulator piezoelectric material covered by two flat electrodes on its two surfaces. Once the NG suffers a vertical mechanical deformation, piezoelectric polarization charges are generated at the two ends of the material. The polarization charge density σ_p can be increased by increasing the applied force, and the electrostatic potential created by the polarization charges is balanced by the flow of electrons between two electrodes through a load. If the induced charge density on the electrode is $\sigma(t)$, the corresponding electric field in the medial is

$$E = (\sigma - \sigma_p)/\varepsilon \quad (20.1)$$

Substituting Eq. (20.1) into Eq. (19.2), the transport Eq. is

$$RA\frac{d\sigma}{dt} + z\frac{\sigma - \sigma_p}{\varepsilon} = 0. \quad (20.2)$$

where A is the area of the electrode, and z is the thickness of the

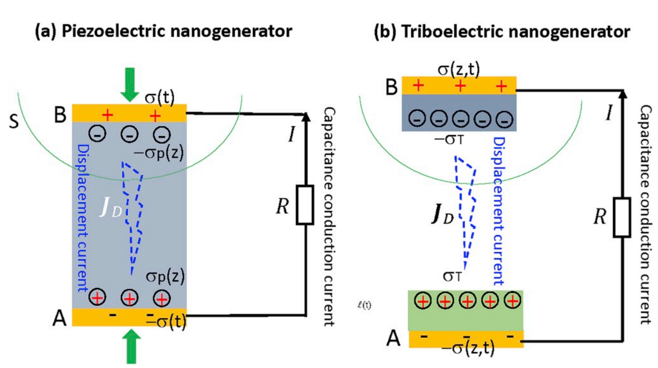

Fig. 3. (a) Thin film based piezoelectric nanogenerators and (b) Contact-separation mode triboelectric nanogenerators.

piezoelectric film. The piezoelectric surface charge density is related to the strain s by

$$\sigma_p = es \quad (20.3)$$

where e is the piezoelectric coefficient and s is the strain. If the thickness variation of the piezoelectric film is ignored and the edge effect is ignored, Eq. (20.2) has a general solution:

$$\sigma = \sigma_p\left[1 - \exp\left(-\frac{z}{RA\varepsilon}t\right)\right] \quad (20.4)$$

Therefore, the output power on the load is:

$$p = \left\{\frac{z\sigma_p}{R\varepsilon}\, exp\left(-\frac{z}{RA\varepsilon}t\right)\right\}^2 R;\ \text{and the total output energy is } E_0 = \frac{Az(\sigma_p)^2}{2\varepsilon}. \quad (20.5)$$

3.4. The output of triboelectric nanogenerators

The scientific term for triboelectrification is called contact-electrification (CE) scientifically, which means that a physical contact between two different materials (even chemically the same type of materials) would be electrically charged after they are separated. But the two have significant difference. *Triboelectrification is a convolution of two processes between tribology and CE, so that they are inseparable. But in science, contact-electrification occurs just by physical contact even without rubbing one material against the other, while tribology refers to the mechanical rubbing between materials. Therefore, we should be clear that CE is a science term, triboelectrification is an engineering term, both have significant difference.*

Although triboelectrification sounds not fancy, but it exists everywhere and anywhere and at all the time, and it has been known for 2600 years. It is probably the most universal phenomenon in our environment and in nature. But there is a debate regarding if CE is due to electrons transfer, ion transfer or even materials species transfer. Recently, we found that CE between two solids is dominated if not exclusively by electron transfer [20]. *Therefore, we can redefined contact electrification to be a quantum mechanical electron and ion transfer process that occurs for any materials, in any states (solid, liquid, gas), in any application environment, and in a wide range of temperature up to ~400 °C.* [20] *Such an effect is universal and is fundamentally unique in nature.* This effect is recently becoming exciting and revived because of its applications for converting mechanical energy into electric power based on the TENG.

To illustrate the application of the theory presented above for TENG, we choose the contact-separation mode as a simple case (Fig. 3b). The TENG is made of two dielectric layers separated by a gap, with electrodes on the top and bottom surfaces of the top and bottom surfaces of the two dielectric layers, respectively. For the TENG, electrostatic charges with opposite signs are generated on the surfaces of two dielectrics after the physical contact. If the two dielectrics have permittivity of ε_1 and ε_2 and thicknesses of d_1 and d_2, respectively, and the triboelectricity introduced surface charge density is $\sigma_T(t)$, and the density of free electrons on surfaces of the electrode is $\sigma(z,t)$. By assuming the planar size of the dielectric film is much larger than the gap distance, and ignoring the field leakage effect at the edge, the electric fields in the two media and in the gap are respectively

$$E_z = -\sigma(z,t)/\varepsilon_1, \quad (21.1)$$

$$E_z = -\sigma(z,t)/\varepsilon_2, \quad (21.2)$$

$$E_z = -(\sigma(z,t) - \sigma_T)/\varepsilon_0. \quad (21.3)$$

The potential drop between the A and B electrodes is

$$\phi_{AB} = -\sigma(z,t)[d_1/\varepsilon_1 + d_2/\varepsilon_2] - H(t)[\sigma(z,t) - \sigma_T]/\varepsilon_0 \quad (22.1)$$

From other hand, from the displacement current and Ohm's law across the load,

$$\phi_{AB} = AJ_D R = AR\frac{\partial \mathbf{D}_z}{\partial t} = AR\frac{\partial \sigma(z,t)}{\partial t} \tag{22.2}$$

The transport equation across an external load R is thus:

$$AR\frac{\partial \sigma(z,t)}{\partial t} = -\sigma(z,t)[d_1/\varepsilon_1 + d_2/\varepsilon_2] - H(t)[\sigma(z,t) - \sigma_T]\Big/\varepsilon_0 \tag{22.3}$$

e.g.,

$$\frac{\partial \sigma(z,t)}{\partial t} = -\sigma(z,t)\frac{d_1/\varepsilon_1 + d_2/\varepsilon_2 + H(t)/\varepsilon_0}{RA} + \frac{H(t)\sigma_T}{RA\varepsilon_0} \tag{22.4}$$

H is a function of time and it is determined by the rate at which the two dielectrics are contacted. This is a general transport equation that can be solved analytically and numerically [8].

From Eq. (22.3), *the choice of the size and geometry of the dielectric layers and electrodes have to be optimized in order to ensure "enough" leakage field for maximize the output power of the TENG in real system.* We need to choose the right size and geometrical arrangement, even with holes punched on electrode, in order to maximize the output of the TENG as a function of other parameters such as the thickness of the dielectric layer and the gap distance. Using Eq. (19.2) we can optimize the output performance by calculating the output power for the total integrated devices, which is important for designing high efficient TENG. This is an important point that one has to do for TENG design.

Under short-circuit condition $\phi_{AB} = 0$, Eq. (22.1) gives:

$$\sigma(z,t) = \frac{H\sigma_T}{d_1\varepsilon_0/\varepsilon_1 + d_2\varepsilon_0/\varepsilon_2 + z} \tag{23.1}$$

The corresponding displacement current density at short-circuit case is

$$\mathbf{J}_D = \frac{\partial \mathbf{D}_z}{\partial t} = \frac{\partial \sigma(z,t)}{\partial t} = \sigma_T\frac{dH}{dt}\frac{d_1\varepsilon_0/\varepsilon_1 + d_2\varepsilon_0/\varepsilon_2}{[d_1\varepsilon_0/\varepsilon_1 + d_2\varepsilon_0/\varepsilon_2 + z]^2} + \frac{d\sigma_T}{dt}\frac{H}{d_1\varepsilon_0/\varepsilon_1 + d_2\varepsilon_0/\varepsilon_2 + z} \tag{23.2}$$

In Eq. (23.2), the first term means that the magnitude of the displacement current is proportional to the speed at which the two media contact/separate (dH/dt); the second term is related to the rate at which the surface charge density building up, which usually drops out after the TENG runs for about 10 cycles, thus,

$$\mathbf{J}_D \approx \sigma_T\frac{dH}{dt}\frac{d_1\varepsilon_0/\varepsilon_1 + d_2\varepsilon_0/\varepsilon_2}{[d_1\varepsilon_0/\varepsilon_1 + d_2\varepsilon_0/\varepsilon_2 + z]^2} \tag{23.3}$$

3.5. *Electric potential for contact-separation mode TENG*

We now use Eq. (16.1) to calculate the potential distribution in space for the contact-separation mode TENG.

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon}\int \frac{\sigma(\mathbf{r}', t) + \sigma_T(\mathbf{r}', t')}{|\mathbf{r} - \mathbf{r}'|}ds' \tag{24.1}$$

We can easily write down the current density for a plate with width W and length L in the contact-separation mode TENG (Fig. 4a):

$$\sigma(\mathbf{r}, t) \approx \begin{cases} -\sigma\delta(z + d_2)\ if\ -\frac{W}{2} \le x \le \frac{W}{2}\ and\ -\frac{L}{2} \le y \le \frac{L}{2} \\ \sigma\delta(z - H(t) - d_1)\ -\frac{W}{2} \le x \le \frac{W}{2}\ and\ -\frac{L}{2} \le y \le \frac{L}{2}; \\ 0,\ otherwise. \end{cases} \tag{24.2}$$

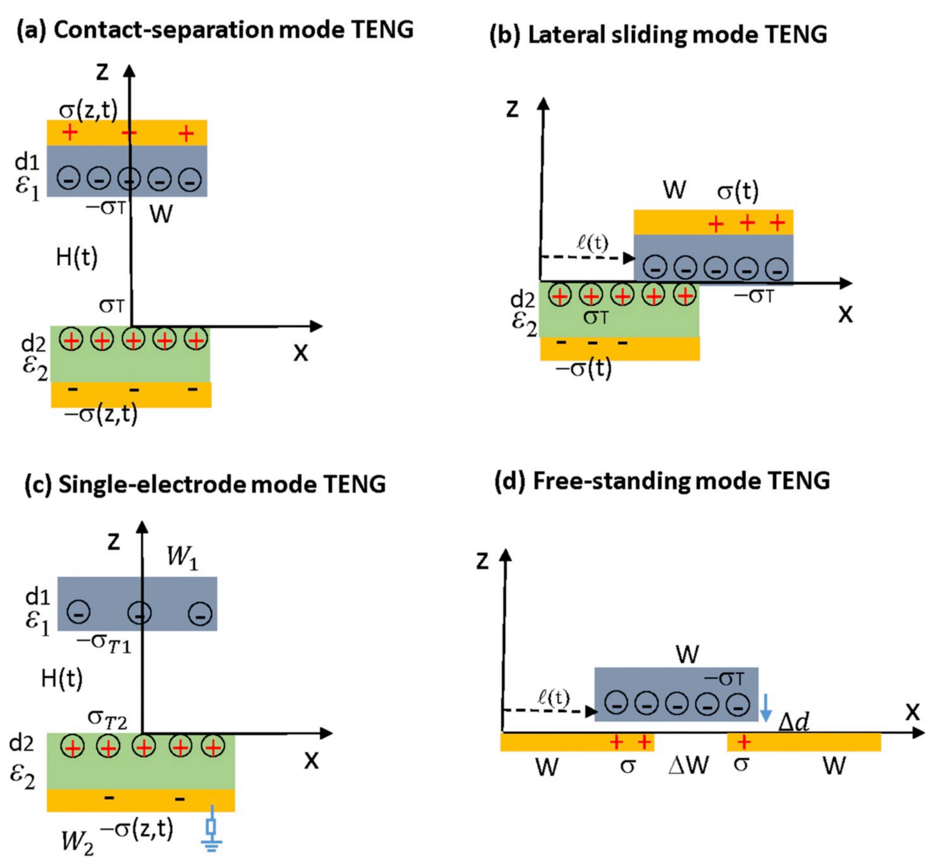

Fig. 4. Coordination system and mathematical parameters defined for describing (a) contact-separation, (b) lateral-sliding mode, (c) single-electrode mode and (d) free-standing mode TENGs, respectively. The conduction line for the external load R are not drawn here for clarity.

$$\sigma_T(\mathbf{r},\ t) = \begin{cases} \sigma_T\delta(z), & if\ -\frac{W}{2} \le x \le \frac{W}{2}\ and\ -\frac{L}{2} \le y \le \frac{L}{2} \\ -\sigma_T\delta(z-H(t)), & if\ -\frac{W}{2} \le x \le \frac{W}{2}\ and\ -\frac{L}{2} \le y \le \frac{L}{2}; \\ 0, & otherwise. \end{cases} \tag{24.3}$$

The potential distribution in space is:

$$\begin{aligned} \phi(\mathbf{r},t) = {} & \frac{1}{4\pi\varepsilon}\int_{-w/2}^{w/2} dx' \int_{-L/2}^{L/2} dy' \Big\{ -\sigma\left[(x-x')^2+(y-y')^2+(z+d_2)^2\right]^{-1/2} \\ & +\sigma\left[(x-x')^2+(y-y')^2+(z-H(t)-d_1)^2\right]^{-1/2} \\ & +\sigma_T\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2} - \sigma_T\left[(x-x')^2+(y-y')^2+(z-H(t))^2\right]^{-1/2}\Big\} \\ = {} & \frac{1}{4\pi\varepsilon}\int_{-w/2}^{w/2} dx' \Bigg\{ -\sigma\ell n\left\{\frac{\left|y+L/2+\left[(x-x')^2+\left(y+L/2\right)^2+\left(z+d_2\right)^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+\left(y-L/2\right)^2+\left(z+d_2\right)^2\right]^{1/2}\right|}\right\} \\ & +\sigma\ell n\left\{\frac{\left|y+L/2+\left[(x-x')^2+\left(y+L/2\right)^2+\left(z-H(t)-d_1\right)^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+\left(y-L/2\right)^2+\left(z-H(t)-d_1\right)^2\right]^{1/2}\right|}\right\} \\ & +\sigma_T\ell n\left\{\frac{\left|y+L/2+\left[(x-x')^2+\left(y+L/2\right)^2+z^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+\left(y-L/2\right)^2+z^2\right]^{1/2}\right|}\right\} \\ & -\sigma_T\ell n\left\{\frac{\left|y+L/2+\left[(x-x')^2+\left(y+L/2\right)^2+\left(z-H(t)\right)^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+\left(y-L/2\right)^2+\left(z-H(t)\right)^2\right]^{1/2}\right|}\right\}\Bigg\} \end{aligned} \tag{24.4}$$

If we ignore film thickness d_1 and d_2 in comparison to the size of the dielectric films,

$$\begin{aligned} \phi(\mathbf{r},t) \approx {} & \frac{\sigma_T-\sigma}{4\pi\varepsilon}\int_{-w/2}^{w/2} dx' \int_{-L/2}^{L/2} dy' \Big\{\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2} \\ & -\left[(x-x')^2+(y-y')^2+(z-H(t))^2\right]^{-1/2}\Big\} \\ = {} & \frac{\sigma_T-\sigma}{4\pi\varepsilon}\int_{-w/2}^{w/2} dx' \Bigg\{\ell n\left\{\frac{\left|y+L/2+\left[(x-x')^2+(y+L/2)^2+z^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+(y-L/2)^2+z^2\right]^{1/2}\right|}\right\} \\ & -\ell n\left\{\frac{\left|y+L/2+\left[(x-x')^2+(y+L/2)^2+(z-H(t))^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+(y-L/2)^2+(z-H(t))^2\right]^{1/2}\right|}\right\}\Bigg\} \end{aligned} \tag{24.5}$$

The transport behavior of the TENG is by calculating the potential drop between the two electrodes using Eq. (24.4) or Eq. (24.5) and then substitute it into Eq. (19.2) and solve it numerically [8].

3.6. Electric potential for lateral-sliding mode TENG

For the lateral-sliding mode TENG, we can easily write down the current density for a plate with width W (sliding direction) and length L, with a sliding distance of $\ell(t)$ ($\ell(t) \le W$) (Fig. 4b):

$$\sigma(\mathbf{r},\ t) = \begin{cases} -\sigma\delta(z+d_2), & if\ 0 \le x \le l(t)\ and\ -\frac{L}{2} \le y \le \frac{L}{2}; \\ \sigma\delta(z-d_1), & if\ w \le x \le l(t)+w,\ and\ -\frac{L}{2} \le y \le \frac{L}{2}; \\ 0, & otherwise. \end{cases} \tag{25.1}$$

$$\sigma_T(\mathbf{r},\ t) = \begin{cases} \sigma_T\delta(z), & if\ 0 \le x \le l(t) and\ -\frac{L}{2} \le y \le \frac{L}{2}; \\ -\sigma_T\ \delta(z), & if\ w \le x \le l(t)+w,\ and\ -\frac{L}{2} \le y \le \frac{L}{2}; \\ 0, & otherwise. \end{cases} \tag{25.2}$$

Here we purpose to write a delta function to specify the distribution of charges on surfaces.

$$\begin{aligned} \phi(\mathbf{r},t) = {} & \frac{1}{4\pi\varepsilon}\Bigg\{\int_0^{\ell(t)} dx' \int_{-L/2}^{L/2} dy' \Big\{ \\ & -\sigma\left[(x-x')^2+(y-y')^2+(z+d_2)^2\right]^{-1/2} \\ & +\sigma_T\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2}\Big\} \\ & +\int_W^{\ell(t)+W} dx' \int_{-L/2}^{L/2} dy' \Big\{\sigma\left[(x-x')^2+(y-y')^2+(z-d_1)^2\right]^{-1/2} \\ & -\sigma_T\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2}\Big\}\Bigg\} \end{aligned} \tag{26.1}$$

If the film thickness is so small for sliding mode in comparison to film size and ignore the free charge distribution at the edge,

$$\begin{aligned} \phi(\mathbf{r},t) = {} & \frac{\sigma_T-\sigma}{4\pi\varepsilon}\int_{-L/2}^{L/2} dy' \Big\{\int_0^{\ell(t)} dx' \left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2} \\ & -\int_W^{\ell(t)+W} dx' \left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2}\Big\} \\ = {} & \frac{\sigma_T-\sigma}{4\pi\varepsilon}\Bigg\{\int_{-L/2}^{L/2} dy' \Big\{\ell n\left\{\frac{\left|x+\ell(t)+\left[(x+\ell(t))^2+(y-y')^2+z^2\right]^{1/2}\right|}{\left|x+\left[x^2+(y-y')^2+z^2\right]^{1/2}\right|}\right\} \\ & -\ell n\left\{\frac{\left|x+\ell(t)+W+\left[(x+\ell(t)+W)^2+(y-y')^2+z^2\right]^{1/2}\right|}{\left|x+W+\left[(x+W)^2+(y-y')^2+z^2\right]^{1/2}\right|}\right\}\Big\}\Bigg\} \end{aligned} \tag{26.2}$$

The transport behavior of the TENG is by calculating the potential drop between the two electrodes using Eq. (26.1) or Eq. (26.2) and then substitute it into Eq. (19.2) and solve it numerically [9].

3.7. Electric potential for single-electrode mode TENG

For single-electrode mode TENG (Fig. 4c), if we ignore the edge effect and assume the gap distance is much smaller than the size of the dielectric layer, the distribution of charges on the surfaces are:

$$\sigma(\mathbf{r},\ t\ )\ \approx \begin{cases} -\sigma\delta(z+d_2)\ if\ -\frac{W_2}{2}\le x\le\frac{W_2}{2}\ and\ -\frac{L}{2}\le y\le\frac{L}{2}; \\ 0,\ otherwise. \end{cases} \quad (27.1)$$

$$\sigma_T(\mathbf{r},\ t\ )\ = \begin{cases} \sigma_{T2}\delta(z),\ \ if\ -\frac{W_2}{2}\le x\le\frac{W_2}{2}\ and\ -\frac{L}{2}\le y\le\frac{L}{2} \\ -\sigma_{T1}\delta(z-H(t)),\ \ if -\frac{W_1}{2}\le x\le\frac{W_1}{2}\ and\ -\frac{L}{2}\le y\le\frac{L}{2}; \\ 0,\ otherwise. \end{cases} \quad (27.2)$$

The potential drop is

$$\phi(\mathbf{r},\mathrm{t}) = \frac{1}{4\pi\varepsilon}\int_{-W_2/2}^{W_2/2}\mathrm{dx'}\int_{-L/2}^{L/2}\mathrm{dy'}\left\{-\sigma\left[(x-x')^2+(y-y')^2+(z+d_2)^{2-1/2}\right]\right.$$
$$\left.+\sigma_{T2}\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2}\right\} - \frac{1}{4\pi\varepsilon}\int_{-W_1/2}^{W_1/2}\mathrm{dx'}\int_{-L/2}^{L/2}\mathrm{dy'}\left\{\sigma_{T1}\left[(x-x')^2+(y-y')^2+(z-H(t))^2\right]^{-1/2}\right\}$$
$$= -\frac{\sigma}{4\pi\varepsilon}\int_{-W_2/2}^{W_2/2}\mathrm{dx'}\ \ell\mathrm{n}\left\{\frac{\left|y+L/2+\left[(x-x')^2+(y+L/2)^2+(z+d_2)^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+(y-L/2)^2+(z+d_2)^2\right]^{1/2}\right|}\right\}$$
$$+\frac{\sigma_{T2}}{4\pi\varepsilon}\int_{-W_2/2}^{W_2/2}\mathrm{dx'}\ \ell\mathrm{n}\left\{\frac{\left|y+L/2+\left[(x-x')^2+(y+L/2)^2+z^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+(y-L/2)^2+z^2\right]^{1/2}\right|}\right\}$$
$$-\frac{\sigma_{T1}}{4\pi\varepsilon}\int_{-W_1/2}^{W_1/2}\mathrm{dx'}\ \ell\mathrm{n}\left\{\frac{\left|y+L/2+\left[(x-x')^2+(y+L/2)^2+(z-H(t))^2\right]^{1/2}\right|}{\left|y-L/2+\left[(x-x')^2+(y-L/2)^2+(z-H(t))^2\right]^{1/2}\right|}\right\} \quad (27.3)$$

The transport behavior of the TENG is by calculating the potential drop between the electrode and the ground using Eq. (27.3) and then add the potential drop across the load R, then substitute it into Eq. (19.2) and solve it numerically [10].

3.8. Electric potential for free-standing mode TENG

For the free-standing mode TENG, if the gap between the top dielectric and the bottom electrodes is Δd and assume the gap is rather small so that we can ignore the edge effect, the charge distribution is

$$\sigma(\mathbf{r},\ t\ )\ = \begin{cases} \sigma\delta(z),\ \ if\ l(t)\le x\le w\ and\ -\frac{L}{2}\le y\le\frac{L}{2}; \\ \sigma\delta(z),\ \ if\ w+\Delta w\le x\le l(t)+w,\ and\ -\frac{L}{2}\le y\le\frac{L}{2}; \\ 0,\ otherwise. \end{cases} \quad (28.1)$$

$$\sigma_T(\mathbf{r},\ t\ )\ = \begin{cases} -\sigma_T\delta(z-\Delta d),\ \ if\ l(t)\le x\le l(t)+w\ and\ -\frac{L}{2}\le y\le\frac{L}{2}; \\ 0,\ otherwise. \end{cases} \quad (28.2)$$

If the gap distance Δd is negligible in comparison to the size of the dielectric media, the potential drop is thus

$$\phi(\mathrm{r,t})\approx\frac{1}{4\pi\varepsilon}\left\{\int_{\ell(\mathrm{t})}^{\mathrm{w}}\mathrm{dx'}\int_{-L/2}^{L/2}\mathrm{dy'}\left\{\sigma\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2}\right\}\right.$$
$$+\int_{\mathrm{W+\Delta W}}^{\ell(\mathrm{t})+\mathrm{W}}\mathrm{dx'}\int_{-L/2}^{L/2}\mathrm{dy'}\left\{\sigma\left[(x-x')^2+(y-y')^2+z^2\right]^{-1/2}\right\}$$
$$\left.-\int_{\ell(\mathrm{t})}^{\ell(\mathrm{t})+\mathrm{W}}\mathrm{dx'}\int_{-L/2}^{L/2}\mathrm{dy'}\left\{\sigma_T\left[(x-x')^2+(y-y')^2+(z-\Delta d)^2\right]^{-1/2}\right\}\right\}$$
$$=\frac{\sigma}{4\pi\varepsilon}\left\{\int_{-L/2}^{L/2}\mathrm{dy'}\left\{\ell\mathrm{n}\left\{\frac{\left|x+w+\left[(x+w)^2+(y-y')^2+z^2\right]^{1/2}\right|}{\left|x+\ell(t)+\left[(x+\ell(t))^2+(y-y')^2+z^2\right]^{1/2}\right|}\right\}\right.\right.$$
$$\left.\left.+\ell\mathrm{n}\left\{\frac{\left|x+\ell(t)+W+\left[(x+\ell(t)+W)^2+(y-y')^2+z^2\right]^{1/2}\right|}{\left|x+W+\Delta W+\left[(x+W+\Delta W)^2+(y-y')^2+z^2\right]^{1/2}\right|}\right\}\right\}\right\}$$
$$-\frac{\sigma_T}{4\pi\varepsilon}\int_{-L/2}^{L/2}\mathrm{dy'}\ell\mathrm{n}\left\{\frac{\left|x+\ell(t)+W+\left[(x+\ell(t)+W)^2+(y-y')^2+(z-\Delta d)^2\right]^{1/2}\right|}{\left|x+\ell(t)+\left[(x+\ell(t))^2+(y-y')^2+(z-\Delta d)^2\right]^{1/2}\right|}\right\} \quad (28.3)$$

The transport behavior of the TENG is by calculating the potential drop between the two electrodes using Eq. (28.3) and then substitute it into Eq. (19.2) and solve it numerically [10].

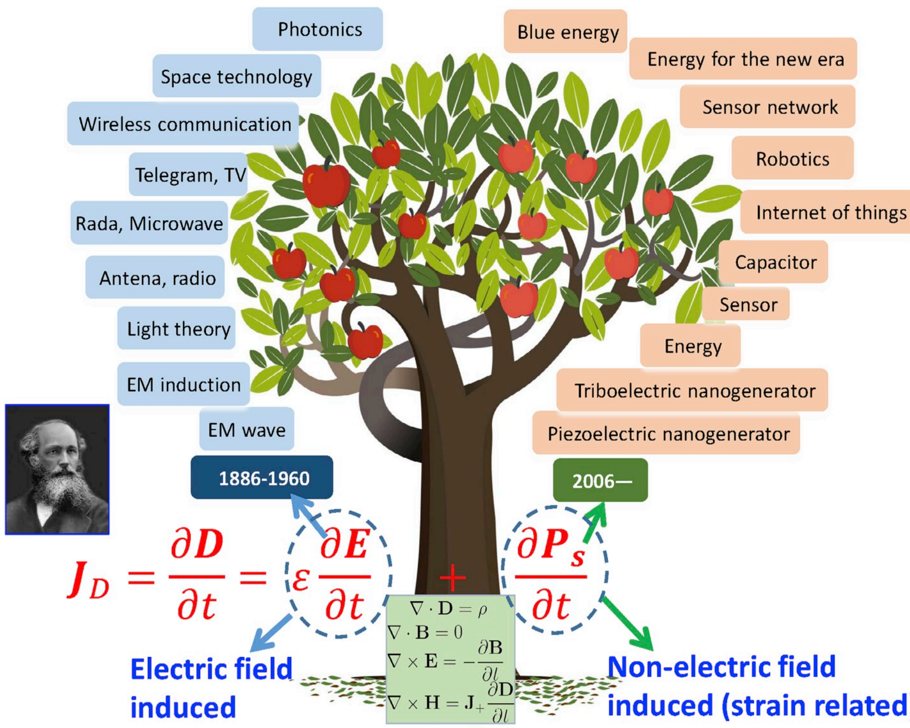

Fig. 5. A tree idea to illustrate the newly revised Maxwell's displacement current: the first term $\varepsilon\partial \mathbf{E}/\partial t$ is responsible for the electromagnetic waves theory; and the newly added term due to $\partial \mathbf{P_s}/\partial t$ is the applications of Maxwell's equations in energy and sensors, which are the nanogenerators.

4. Technology projections from Maxwell's displacement current

From above, we know that the theoretical origin of nanogenerators is the Maxwell's displacement current. The major fundamental science, technologies and practical impacts derived from the two components of the Maxwell's displacement current are presented in Fig. 5. In fact, the first component of displacement current $\varepsilon\frac{\partial \mathbf{E}}{\partial t}$ proposed by Maxwell gives the birth of electromagnetic wave theory, and the electromagnetic induction causes the emergence of antenna, radio, telegram, TV, Radar, microwave, wireless communication, and space technology from 1886. The electromagnetic unification produces the theory of light, laying the physical theory foundation for the invention of laser and development of photonics. In addition, the control and navigation of airplane, shipping, and spacecraft, as well as the technology progress of the electric power and microelectronics industry, cannot be separated from Maxwell's equations. In addition, Maxwell's equations not only predicted the speed of light, but also satisfy the Lorentz transformation for special relativity. It is because Maxwell who inspired of Einstein to start the work of unifying the four forces in nature: electromagnetism, weak interaction, strong interaction and gravity. Therefore, the first component of displacement current has driven the development of the world in communication and laser technology in the last century.

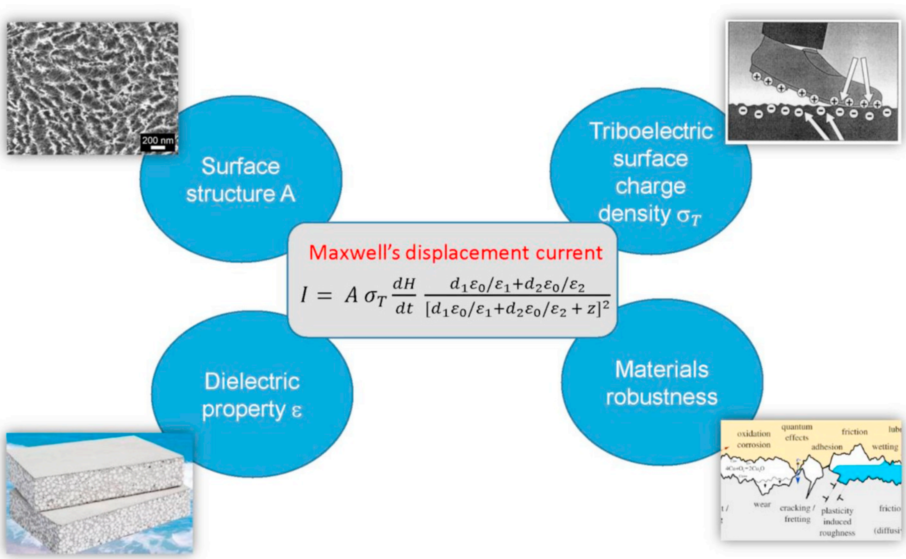

Fig. 6. Using the theoretical result of displacement for contact-separation mode TENG, we show here the requirements and research directions for materials in order to maximize the output of TENG.

In parallel, the second term $\frac{\partial \mathbf{P}_s}{\partial t}$ first proposed by Wang [4] by including the non-electric field induced polarization in the displacement current from mechanical triggering set the foundation for the nanogenerators. Nanogenerators are referred to as the energy for the new era – the era of internet of things and sensor networks [6,19]. Adding a term of $\frac{\partial \mathbf{P}_s}{\partial t}$ in the displacement current and thus in the Maxwell's equations extends their applications to energy! Our nanogenerators for energy could have extensive applications in IoT, sensor networks, blue energy and even big data which will impact the world for the future. The nanogenerators could be regarded as another important application of Maxwell's equations in energy and sensors after the electromagnetic wave theory and technology. For the foreseeable future, the "tree" idea presented in Fig. 5 is expected to grow stronger, taller and larger, which possibly leads to technological breakthroughs that is expected to impact human society in large.

Finally, one can elaborate the materials required for TENG (Fig. 6). The displacement current is related to materials permittivity, triboelectric charge density and surface morphology. Materials with high stability and robustness are important for extending the life time of TENG. We anticipate that huge amount of materials optimization is required following Fig. 6 to enhance the performance of TENG, which opens a new direction for materials scientists and chemists. But such work has not been started yet.

5. Conclusion

In this paper, by introducing a non-electric field induced polarization term $\mathbf{P}_s$ in the Maxwell's equations, a systematic theory has been derived for describing the electromagnetic dynamics of nanogenerators from the first principle point of view, from which the output of NGs can be quantified. Furthermore, the electromagnetic radiation from nanogenerator systems could be calculated, which may be observable if the operation frequency is high. This innovative extension of $\mathbf{P}_s$ in the displacement vector $\mathbf{D}$ opens the application of Maxwell's equations in energy and sensors, which is a new branch of Maxwell's theory besides electromagnetic waves. With the establishment of this general theory, we anticipate that it will serve as the theoretical foundation for advancing the science, technology and even applications of nanogenerators in energy and sensors.

Furthermore, analytical solutions are presented for piezoelectric NG and TENGs, and suggestions are made on how to improve the performance of TENG. Directions for developing TENG related materials are also elaborated. Based on our study, we have defined contact electrification as quantum mechanical electron and/or ion transfer processes that occur for any materials, in any states (solid, liquid, gas), in any application environment, and in a wide range of temperatures.

Declaration of competing interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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